45° to the axis, for in that direction the shearing stress is a maximum. From this it would seem that the resistance overcome at rupture is the resistance of the steel to shear.

Experiments were made to see whether the resistance of steel to direct shearing bore to its resistance to direct tension the ratio required by the above theory; since the greatest shearing stress is equal to one-half the longitudinal stress, we should expect to find the resistance to direct shearing equal to one-half of the resistance to direct tension.

A series of experiments were made with the result that the ultimate resistance to direct shearing was within, on the average, 3 per cent. of the half of that to direct tension.

The appearance of the fracture of steel bars is next discussed. It would appear that when the stress is uniformly distributed in the neighbourhood of the ruptured section, the fracture is at 45° to the axis, the bar having sheared along that plane which is a plane of least resistance to shear. The tendency to rupture along a plane of shear may be masked by a non-uniform distribution of stress.

Two plates of photographs are added, showing examples of steel bars broken by shearing under longitudinal stress.

IV. "Measurements of the Amount of Oil necessary in order to check the Motions of Camphor upon Water." By LORD RAYLEIGH, Sec. R.S. Received March 10, 1890.

The motion upon the surface of water of small camphor scrapings, a phenomenon which had puzzled several generations of inquirers, was satisfactorily explained by Van der Mensbrugghe,* as due to the diminished surface-tension of water impregnated with that body. In order that the rotations may be lively, it is imperative, as was well shown by Mr. Tomlinson, that the utmost cleanliness be observed. It is a good plan to submit the internal surface of the vessel to a preliminary treatment with strong sulphuric acid. A touch of the finger is usually sufficient to arrest the movements by communicating to the surface of the water a film of grease. When the surface-tension is thus lowered, the differences due to varying degrees of dissolved camphor are no longer sufficient to produce the effect.

It is evident at once that the quantity of grease required is excessively small, so small that under the ordinary conditions of experiment it would seem likely to elude our methods of measurement. In view, however, of the great interest which attaches to the determination of molecular magnitudes, the matter seemed well worthy of investigation; and I have found that by sufficiently increasing the water

* 'Mémoires Couronnés' (4to) of the Belgian Academy, vol. 34, 1869.

surface the quantities of grease required may be brought easily within the scope of a sensitive balance.

In the present experiments the only grease tried is olive oil. It is desirable that the material which is to be spread out into so thin a film should be insoluble, involatile, and not readily oxidised, requirements which greatly limit the choice.

Passing over some preliminary trials, I will now describe the procedure by which the density of the oil film necessary for the purpose was determined. The water was contained in a sponge-bath of extra size, and was supplied to a small depth by means of an india-rubber pipe in connexion with the tap. The diameter of the circular surface thus obtained was 84 cm. (33″). A short length of fine platinum wire, conveniently shaped, held the oil. After each operation it was cleaned by heating to redness, and counterpoised in the balance. A small quantity of oil was then communicated, and determined by the difference of readings. Two releasements of the beam were tried in each condition of the wire, and the deduced weights of oil appeared usually to be accurate to $\frac{1}{20}$ milligram at least. When all is ready, camphor scrapings are deposited upon the water at two or three places widely removed from one another, and enter at once into vigorous movement. At this stage the oiled extremity of the wire is brought cautiously down so as to touch the water. The oil film advances rapidly across the surface, pushing before it any dust or camphor fragments which it may encounter. The surface of the liquid is then brought into contact with all those parts of the wire upon which oil may be present, so as to ensure the thorough removal of the latter. In two or three cases it was verified by trial that the residual oil was incompetent to stop camphor motions upon a surface including only a few square inches.

The manner in which the results are exhibited will be best explained by giving the details of the calculation for a single case, *e.g.*, the second of December 17. Here 0·81 milligram of oil was found to be very nearly enough to stop the movements. The volume of oil in cubic centimetres is deduced by dividing 0·00081 by the sp. gr., viz., 0·9. The surface over which this volume of oil is spread is

$$\tfrac{1}{4}\pi \times 84^2 \text{ square centimetres};$$

so that the thickness of the oil film, calculated as if its density were the same as in more normal states of aggregation, is

$$\frac{0{\cdot}00081}{0{\cdot}9 \times \frac{1}{4}\pi \times 84^2} = \frac{1{\cdot}63}{10^7} \text{ cm.},$$

or 1·63 micro-millimetres. Other results, obtained as will be seen at considerable intervals of time, are collected in the Table. For conve-

nience of comparison they are arranged, not in order of date, but in order of densities of film.

The sharpest test of the quantity of oil appeared to occur when the motions were nearly, but not quite, stopped. There may be some little uncertainty as to the precise standard indicated by "nearly enough," and it may have varied slightly upon different occasions. But the results are quite distinct, and under the circumstances very accordant. The thickness of oil required to take the life out of the camphor movements lies between one and two millionths of a millimetre, and may be estimated with some precision at 1·6 micro-millimetre. Preliminary results from a water surface of less area are quite in harmony.

For purposes of comparison it will be interesting to note that the thickness of the black parts of soap films was found by Messrs. Reinold and Rücker to be 12 micro-millimetres.

A Sample of Oil, somewhat decolorised by exposure.

Date.	Weight of oil.	Calculated thickness of film.	Effect upon camphor fragments.
Dec. 17...	0·40 mg.	0·81	No distinct effect.
Jan. 11 ..	0·52	1·06	Barely perceptible.
Jan. 14...	0·65	1·32	Not quite enough.
Dec. 20...	0·78	1·58	Nearly enough.
Jan. 11...	0·78	1·58	Just enough.
Dec. 17...	0·81	1·63	Just about enough.
Dec. 18...	0·83	1·68	Nearly enough.
Jan. 22...	0·84	1·70	About enough.
Dec. 18...	0·95	1·92	Just enough.
Dec. 17...	0·99	2·00	All movements very nearly stopped.
Dec. 20...	1·31	2·65	Fully enough.
A fresh Sample.			
Jan. 28...	0·63	1·28	Barely perceptible.
Jan. 28...	1·06	2·14	Just enough.

An important question presents itself as to how far these water surfaces may be supposed to have been clean to begin with. I believe that all ordinary water surfaces are sensibly contaminated; but the agreement of the results in the Table seems to render it probable that the initial film was not comparable with that purposely contributed. Indeed, the difficulties of the experiments proved to be less than had been expected. Even a twenty-four hours' exposure to the air of the

laboratory* does not usually render a water surface unfit to exhibit the camphor movements.

The thickness of the oil films here investigated is of course much below the range of the forces of cohesion; and thus the tension of the oily surface may be expected to differ from that due to a complete film, and obtained by addition of the tensions of a water-oil surface and of an oil-air surface. The precise determination of the tension of oily surfaces is not an easy matter. A capillary tube is hardly available, as there would be no security that the degree of contamination within the tube was the same as outside. Better results may be obtained from the rise of liquid between two parallel plates. Two such plates of glass, separated at the corners by thin sheet metal, and pressed together near the centre, dipped into the bath. In one experiment of this kind the height of the water when clean was measured by 62. When a small quantity of oil, about sufficient to stop the camphor motions, was communicated to the surface of the water, it spread also over the surface included between the plates, and the height was depressed to 48. Further additions of oil, even in considerable quantity, only depressed the level to 38.

The effect of a small quantity of oleate of soda is much greater. By this agent the height was depressed to 24, which shows that the tension of a surface of soapy water is much less than the combined tensions of a water-oil and of an oil-air surface. According to Quincke, these latter tensions are respectively 2·1 and 3·8, giving by addition 5·9; that of a water-air surface being 8·3. When soapy water is substituted for clean, the last number certainly falls to less than half its value, and therefore much below 5·9.

V. "On the Stability of a Rotating Spheroid of Perfect Liquid." By G. H. Bryan. Communicated by Professor G. H. Darwin, F.R.S. Received March 12, 1890.

1. In my communication on "The Waves on a Rotating Liquid Spheroid of Finite Ellipticity,"† I stated that it did not appear possible to give a complete investigation of the criteria of stability of Maclaurin's spheroid when the liquid forming it is free from all traces of viscosity, and equilibrium is liable to be broken by a disturbance of a perfectly general character. As the problem in question appeared to be one of considerable interest, I have, since writing the above paper, put the question to the test of numerical calculation in the case of the simpler types of disturbance, and the results thus obtained have been such as to allow of extension to a perfectly general disturbance.

* In the country.

† 'Phil. Trans.,' A, 1889, p. 187.